Monty Hall Simulation

Justin Burruss

9/19/2021

Background

In the Monty Hall game a prize is hidden at random behind one of three doors. The player first guesses one door at random, then one of the other two doors is opened to reveal that the prize is not there. The player may then choose to keep their initial guess or to switch their choice to the unopened door. Should the player switch doors or keep their initial guess?

Most people if asked (and not given the chance to think it though carefully) will quickly answer that one door is as good as the other, and that the player may as well keep their initial guess. This is a classic example of where intuition about probabilities can break down. The truth is that the player is twice as likely to win if they always switch doors.

Initially one closed door has the same probability of concealing the prize as any other closed door, and the player has a 1/3 chance of winning and a 2/3 chance of losing. Thinking it through, though, given an incorrect first choice (2/3 probability), the second remaining incorrect door has a 100% chance of being opened, leaving a probability that the remaining closed door conceals the prize of 100% x 2/3 = 2/3. Thus, the player should always switch. The initial door has only 1/3 probability of success while the other door has 2/3 probability of success.

Thinking about a third alternative, if after one incorrect door was opened we were to make a fresh random 50/50 guess between the two still-concealed doors, we would expect success probability of 1/2.

So in theory, we have three strategies with three probabilities of success: keep the initial guess (1/3 success), re-guess at random (1/2 success), and always switch (2/3 success). These truths may contradict intuition, so can we simulate a large number of games and examine the actual outcomes?

Simulating three Monty Hall strategies

We repeatedly simulate Monty Hall games, outputting the result of three strategies: sticking with the initial guess, re-guessing, or always switching.

```r
library(data.table)

fGuess <- function() {
    actual <- sample(1:3, 1)
    firstGuess <- sample(1:3, 1)
    removed <- (c(1:3)[-c(firstGuess, actual)])[sample.int(length(c(1:3)[-c(firstGuess,
        actual)]), 1)]
    switchGuess <- c(1:3)[-c(firstGuess, removed)]
    reGuess <- sample(c(switchGuess, firstGuess), 1)
    return(data.table(FirstGuess = (firstGuess == actual), ReGuess = (reGuess ==
        actual), SwitchGuess = (switchGuess == actual)))
```

```r
}

t0 <- Sys.time()

trials <- data.table(RunId = seq(1, 10^5, 1))
trials <- trials[, fGuess(), by = RunId]

t1 <- Sys.time()

# Display our wall clock run time
t1 - t0
```

```
## Time difference of 11.26592 secs
```

```r
# How successful was the strategy of sticking with the
# initial guess? In theory this should be 1/3.
sum(trials$FirstGuess/nrow(trials))
```

```
## [1] 0.33221
```

```r
# How successful was the strategy of re-guessing? In theory
# this should be 1/2.
sum(trials$ReGuess/nrow(trials))
```

```
## [1] 0.49878
```

```r
# How successful was the strategy of always switching?  In
# theory this should be 2/3.
sum(trials$SwitchGuess/nrow(trials))
```

```
## [1] 0.66779
```

Conclusion

Our analysis of 100,000 simulations finds that observation is consistent with theory: the best strategy is to switch doors (2/3 probability of success).